Unit 5 Lesson 6: Exponents & Logarithms Review

Exponent Laws:

\[ a^m \cdot a^n = a^{m+n} \]  \[ a^{-n} = \frac{1}{a^n} \]

\[ \frac{a^m}{a^n} = a^{m-n} \]  \[ (\frac{a}{b})^{-n} = (\frac{b}{a})^n \]

\[ (a^m)^n = a^{mn} \]  \[ a^{\frac{1}{n}} = \sqrt[n]{a} \]

\[ (ab)^n = a^n b^n \]  \[ a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m \]

\[ \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \]

Logarithmic Rules:

\[ \log_n a + \log_n b = \log_n(ab) \]  \[ \log_n n = 1 \]

\[ \log_n a - \log_n b = \log_n\frac{a}{b} \]  \[ \log_n 1 = 0 \]

\[ \log_n a^b = b \log_n a \]  \[ \log_a b = \frac{\log_n b}{\log_n a} \]

\[ A^B = C \quad \text{means the same as} \quad \log_A C = B \]
Common Logarithms:

- These are logarithms that are base 10
- Shorthand → \( \log x \)

Natural Logarithms:

- These are logarithms that are base \( e \)
  
  \[ e = 2.71828... \]
  
  \( (e \) is an irrational number)\)

- Shorthand → \( \ln x \) or \( \ln x \)

Ex) Evaluate the following.

a) \[ \log_3 27^5 \]
   
   \[ = 5 \log_3 27 \]
   
   \[ = 5 (3) \]
   
   \[ = 15 \]

b) \[ \log_2 384 - \log_2 3 \]
   
   \[ = \log_2 \left( \frac{384}{3} \right) \]
   
   \[ = \log_2 128 \]
   
   \[ = 7 \]

c) \[ \log_5 \frac{1}{\sqrt[4]{125}} \]
   
   \[ = \log_5 125^{-\frac{1}{4}} \]
   
   \[ = -\frac{1}{4} \log_5 125 \]
   
   \[ = -\frac{1}{4} (3) \]
   
   \[ = -\frac{3}{4} \]

d) \[ \ln e^{\log_2 8} \]
   
   \[ = \log_e e^{\log_2 8} \]
   
   \[ = \log_e e^3 \]
   
   \[ = 3 \log_e e \]
   
   \[ = 3 (1) \]
   
   \[ = 3 \]
Ex) If \( \log_2 m = 8 \), evaluate \( \log_2 64m \)

\[
\begin{align*}
\log_2 m &= 8 \\
2^8 &= m \\
256 &= m \\
\log_2 (64 \times 256) &= \log_2 (16384) \\
&= 14
\end{align*}
\]

\[\text{OK} \quad \log_2 64m = \log_2 64 + \log_2 m = 6 + 8 = 14\]

Ex) If \( a = \ln 4 \) and \( b = \ln 7 \), express \( \ln \frac{16807}{64} \) in terms of \( a \) and \( b \).

\[
\begin{align*}
\ln \frac{16807}{64} &= \ln 16807 - \ln 64 \\
&= \ln 7^5 - \ln 4^3 \\
&= 5 \ln 7 - 3 \ln 4 \\
&= 5b - 3a
\end{align*}
\]

Ex) Solve the following for \( x \).

a) \( \log_5 (8x - 16) - \log_5 2 = \log_5 3 + \log_5 (x + 4) \)

(Rewrite each side as a single logarithm then compare what is inside each logarithm.)

\[
\begin{align*}
\log_5 \left( \frac{8x - 16}{2} \right) &= \log_5 3(x + 4) \\
\log_5 (4x - 8) &= \log_5 (3x + 12) \\
4x - 8 &= 3x + 12 \\
x &= 20
\end{align*}
\]
b) \[ \log_7(15x - 3) = 2 + \log_7 3 \]

(When a constant is present, bring all logarithms to one side of the equation and write as a single logarithmic statement. The constant is by itself on the other side of the equation. Convert to an exponential statement and solve.)

\[ \log_7 \left( \frac{15x - 3}{3} \right) = 2 \]

\[ \log_7 (5x - 1) = 2 \]

\[ 7^2 = 5x - 1 \]

\[ 49 = 5x - 1 \]

\[ 50 = 5x \]

\[ x = 10 \]

c) \[ (\log_4 x)^2 - \log_4 x^5 = 14 \]

(When a logarithm is in brackets being squared, replace the said logarithm with an “a”, solve for “a” then substitute the logarithmic statement back for “a” and finish.)

\[ (\log_4 x)^2 - 5 \log_4 x = 14 \]

\[ a = \log_4 x \]

\[ a^2 - 5a = 14 \]

\[ a^2 - 5a - 14 = 0 \]

\[ (a - 7)(a + 2) = 0 \]

\[ a = 7 \quad a = -2 \]

\[ \log_4 x = 7 \quad \text{or} \quad \log_4 x = -2 \]

\[ 4^7 = x \quad 4^{-2} = x \]

\[ x = 16384 \quad x = \frac{1}{16} \]
Ex) Evaluate the following.

a) \[ e^{3\ln 2} = x \]
\[ \log_e x = 3 \ln 2 \]
\[ \ln x = \ln 2^3 \]
\[ x = 8 \]

b) \[ 7^{\log_7(x+4)} = 19 \]
\[ \log_7 19 = \log_7 (x+4) \]
\[ 19 = x+4 \]
\[ x = 15 \]

c) \[ 4 \left( e^{\frac{\ln 169}{2}} \right) = x \]
\[ e^{\frac{\ln 169}{2}} = \frac{x}{4} \]
\[ \ln \left( \frac{x}{4} \right) = \frac{\ln 169}{2} \]
\[ 2 \ln \left( \frac{x}{4} \right) = \ln 169 \]
\[ \ln \frac{x^2}{16} = \ln 169 \]
\[ \Rightarrow \frac{x^2}{16} = 169 \]
\[ x^2 = 16 \times 169 \]
\[ x = 4 \times 13 \]
\[ x = 52 \]

Ex) Solve for \( x \).

\[ 7^{x-2} = 2^{x+5} \]
\[ \log 7^{x-2} = \log 2^{x+5} \]
\[ (x-2) \log 7 = (x+5) \log 2 \]
\[ x \log 7 - 2 \log 7 = x \log 2 + 5 \log 2 \]
\[ x \log 7 - x \log 2 = 5 \log 2 + 2 \log 7 \]
\[ x (\log 7 - \log 2) = 5 \log 2 + 2 \log 7 \]
\[ x = \frac{5 \log 2 + 2 \log 7}{\log 7 - \log 2} \]
\[ x = 5.87 \]